PROVISION FOR RADIATION LOSSES IN MEASURING THE THERMAL CONDUCTIVITIES OF THIN PLATES AND FILMS

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Formulas have been derived for the calculation of the coefficient of thermal conductivity for thin plates and films; these are based on data from the measurements – by a nonsteady method – for the case in which the transfer of heat to the specimen surface is comparable to the flow of heat through the specimen.

In measuring the thermal conductivity of thin specimens such as, for example, plates and films made of semiconductor materials, we must take into consideration the heat losses from the specimen surface, since these, in this case, are comparable to the flow of heat through the specimen.

Let us initially examine how these losses were taken into consideration in a variant of a steady-state measurement method. A test specimen of length l (see Fig. 1) connected to a heater unit is enclosed in a container whose walls are kept at a constant temperature T_0. We measure the thermal conductivity λ as a flow of heat is propagated along a specimen whose upper end has a temperature T_0 = const. The heat flow q through the cross section of the specimen, at its lower end, is equal to q_0-q_1, where $q_0 = ui$ is the power of the heater; q_1 denotes the heat losses from the surface of the heater unit and the quantity of heat removed through the wiring of the thermocouple and the heater. The magnitude of q_0 is chosen so as to satisfy the condition $\Delta T \ll T_0$, where ΔT denotes the temperature difference across the length of the specimen; here we can assume that $\lambda(x)$ = const. The losses of the heat q_1 at the specified temperature $T_1 = T_0 + \Delta T$ are determined experimentally (see below). We can neglect the thermal resistance of the points at which the specimen comes into contact with the heater unit and the container in the case of thin specimens. At all points of the heater unit – made of copper – we can assume the temperature to be constant and equal to T_1.

The heat-conduction equation in this case can be presented in the form

$$\frac{\partial^2 T_x}{\partial x^2} = \frac{C}{\lambda}\frac{\partial T_x}{\partial t} + \frac{Hp}{\lambda S}(T_x - T_0). \tag{1}$$

Since $T_x - T_0 \ll T_0$, we can assume $H(x) \approx$ const. The second term in the right-hand member of (1) reflects the actual heat-transfer conditions all the more exactly, the smaller the fraction of the radiative heat exchange between the specimen and the heater unit relative to the heat exchange between the specimen and the container. This fraction can be reduced to a minimum by using a heater unit of appropriate shape (see Fig. 1). The transfer of heat through the gas surrounding the specimen, in this case, can be kept substantially lower than the radiation losses (given a sufficiently high vacuum in the container). Let us denote $T_x - T_0 = T$ and $Hp/\lambda S = \mu^2$. Since $\partial T_x/\partial t = 0$ in the steady-state regime, we will have

$$\frac{\partial^2 T}{\partial x^2} - \mu^2 T = 0. \tag{2}$$

The solution of this equation for the specified boundary conditions ($T = 0$ for $x = 0$ and $T = T_1 - T_0$ when $x = l$):

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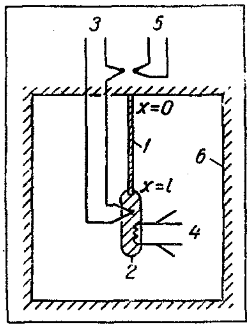

Fig. 1. Measurement circuit: 1) specimen; 2) heater unit; 3) leads for the differential thermocouple to measure $T_1 - T_0$; 4) heater leads and potentiometer leads for the measurement of u; 5) thermocouple leads for the measurement of T_0; 6) container.

$$T = (T_1 - T_0)\frac{\operatorname{sh}\mu x}{\operatorname{sh}\mu l}. \tag{3}$$

Hence

$$q = \lambda S \left.\frac{dT}{dx}\right|_{x=l} = (T_1 - T_0)\,\mu\lambda S \operatorname{cth}\mu l. \tag{4}$$

Having measured $T_1 - T_0$, u, i, H, and q_1 and knowing the constants S and p, we can find λ from Eq. (4).

We can determine the magnitude of q_1 by suspending the heater block within the container from a thin and highly heat-resistant thread. Setting the heater power $u_0 i_0$ in this case so that the heater temperature is T_1, we will find that $q_1 = u_0 i_0$. We can also determine H in similar fashion, attaching the test specimens to the surface of the corresponding unit being heated.

The calculations for the nonsteady method of measuring thermal conductivity, as proposed by Ioffe [1, 2], were performed by Kaganov [3] and by Swan [4]; however, Swan failed entirely to provide for the heat losses from the specimen surfaces and from the unit, while in Kaganov's paper the transfer of heat at the specimen surface is assumed to be substantially smaller than the exchange of heat between the unit and the medium and is not included in the formulas which he derived for the determination of λ.

Let us examine the procedure for measurement by the nonsteady method. The measurement circuit remains as before (see Fig. 1); however, in this case the heater is not switched on. The thermal conductivity of the specimen is determined from the relationship between the temperature difference $T_1 - T_0$ and time. For thin specimens we easily satisfy the condition $C_0/C_1 \ll 1$ and C_1 can always be made larger than C_0 by 2 orders and more.

In the simplest case in which we neglect the transfer of heat involving the specimen, the heater unit, and the ambient medium, and if we also neglect the heat capacity of the specimen, the difference $T_1 - T_0$ as a function of time has the following form

$$T = \Theta_l \exp\left(-\frac{\lambda S}{C_1 l}\, t\right), \tag{5}$$

where $\Theta_l = T$ when $t = 0$.

However, we can use expression (5) to determine the thermal conductivity only within a limited range of temperatures T_0 and specimen thicknesses (the low temperatures and comparatively great thicknesses). In the remaining cases, we must make provision for radiation losses. The condition $C_0/C_1 \ll 1$ substantially simplifies the solution of this problem.

The calculation is reduced to the solution of (1) for the following boundary conditions:

$$\begin{gathered} x = 0;\quad T = 0; \\ x = l;\quad -\frac{\partial T}{\partial t} = \frac{\lambda S}{C_1}\frac{\partial T}{\partial x} + \frac{H_q T}{C_1}. \end{gathered} \tag{6}$$

We will demonstrate that when $C_0/C_1 \ll 1$ the first term in the right-hand member of (1) can be equated to zero. Differentiation of (5) yields

$$\frac{\partial T}{\partial t} \approx -\frac{\lambda S}{C_1 l} T. \tag{7}$$

If the heat transfer at the specimen surface is comparable to the heat flow through the specimen, the order of magnitude for $\partial T/\partial t$ in (1) must be the same as in (7). Consequently,

$$\frac{C}{\lambda}\frac{\partial T}{\partial t} \approx -\frac{C_0}{C_1}\frac{T}{l^2}. \tag{8}$$

The second term in the right-hand member of (1) can be presented in the form

$$\frac{Hpl^2}{\lambda S}\,\frac{T}{l^2}.$$

The quantity $Hpl^2/\lambda S$ under our conditions is of the order of unity or tenths of a fraction; the first term in the right-hand member of (1) can therefore be treated as negligibly small in comparison with the second term. (However, if $Hpl^2/\lambda S \ll 1$, Eq. (1) assumes the form $\partial^2 T/\partial x^2 = 0$.) Physically this means that the temperature distribution over the specimen length at any instant of time (after establishment of the regular regime) differs very little from the distribution in the steady-state case. Equation (1) thus assumes the form

$$\frac{\partial^2 T}{\partial x^2} - \frac{Hp}{\lambda S}T = 0, \tag{9}$$

i.e., we find the solution for Eq. (2):

$$T = \Theta \operatorname{sh} \mu x, \tag{10}$$

however, in this case $\Theta = \Theta(t)$, while the boundary conditions are expressed by relationships (6). From expressions (10) and (6) we obtain the following formula for $T(t)$ when $x = l$:

$$T = A \exp\left[-\left(\frac{\lambda S}{C_1}\mu \operatorname{cth} \mu l + \frac{H_q}{C_1}\right) t\right], \tag{11}$$

where $A = \text{const}$.

The expression for the rate of change in temperature, with consideration of the heat transfer, hence has the form

$$m = \frac{\lambda S}{C_1}\mu \operatorname{cth} \mu l + \frac{H_q}{C_1}. \tag{12}$$

In the special case in which $\mu \gtrsim 1$, expanding $\operatorname{cth} \mu l$ in series, and limiting ourselves to two of its terms, we obtain

$$m = \frac{1}{C_1}\left(\frac{\lambda S}{l} + \frac{Hpl}{3} + H_q\right). \tag{13}$$

Hence we easily determine λ:

$$\lambda = \frac{l}{S}\left(mC_1 - \frac{Hpl}{3} - H_q\right). \tag{14}$$

As with the steady-state measurement method, we can determine the magnitude of H_q by suspending the heater unit within the container from a thread exhibiting great heat resistance. Then $\lambda = 0$ and $Hpl = 0$, and from (14) we obtain

$$H_q = m'C_1.$$

We can also determine H analogously, attaching the test specimen to the surface of a heater unit which exhibits a known heat capacity.

NOTATION

- T_0 is the container temperature;
- T_1 is the heater-unit temperature;
- T_x is the temperature of the point on the specimen at a distance x from the container;
- l is the specimen length;
- S is the cross-sectional area of the specimen;
- λ is the coefficient of thermal conductivity for the specimen;
- C is the volume heat capacity of the specimen;
- C_0 is the heat capacity of the specimen;
- C_1 is the heat capacity of the heater unit;
- H is the heat-transfer coefficient;

p is the perimeter of the specimen's cross section;

q_0 is the power of the heater;

i is the heater current;

u is the heater voltage;

q_1 is the loss of heat from the surface of the heater unit and the loss of heat through the wiring of the thermocouple and the heater;

i_0, u_0 are, respectively, the current and the voltage of the heater for the case in which $\lambda = 0$;

H_q is the heat-transfer coefficient for the heater (radiation and removal of heat through the thermocouple wiring);

m is the rate of temperature change.

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